In the name of God

Engineering Mathematics

(Lecture # 01)

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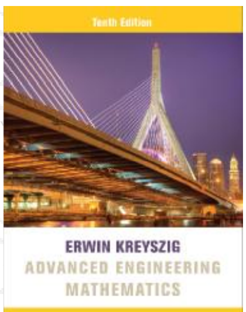
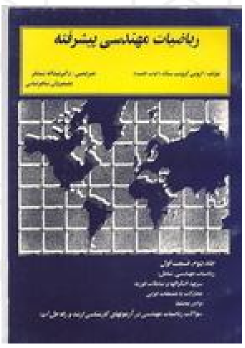
مراجع و منابع

- **Advanced Engineering Mathematics,**
  By: Erwin Kreyszig





- ریاضیات مهندسی پیشرفته
  مترجم: دکتر عبدالله شیدفر

سرفصل مطالب درس

In this course: **We will cover:**

Part I:

- Fourier Analysis.
- Partial Differential Equations (PDEs)

Part II:

- Complex Numbers and Functions.
- Complex Differentiation,
- Complex Integration,
- Power Series, Taylor Series,
- Laurent Series. Residue Integration,
- Conformal Mapping,
- Complex Analysis and Potential Theory. (Maybe!!)

ارزشیابی

- کوئیزها + تمرینها + **فعالیت کلاسی** ٪20
- میانترم ٪30
- پایان ترم ٪50

Do not miss this one!!!

بخش اول:

❖Fourier Analysis

❖Partial Differential Equations

Fourier Analysis

Joseph Fourier
(21 March 1768 – 16 May 1830) was a French mathematician and physicist best known for initiating the investigation of Fourier series and their applications to problems of heat transfer and vibrations.

Fourier, Joseph (1822). *Théorie analytique de la chaleur*. Paris: Firmin Didot Père et Fils.

Fourier Analysis

A function **f(x)** is called a **periodic function** if f (x) is defined for all real x, except possibly at some points, and if there is some positive number p, called a period of , such that

$$f(x + p) = f(x) \quad \text{for all } x. \tag{1}$$

$$f(x + np) = f(x) \quad \text{for all } x. \tag{2}$$

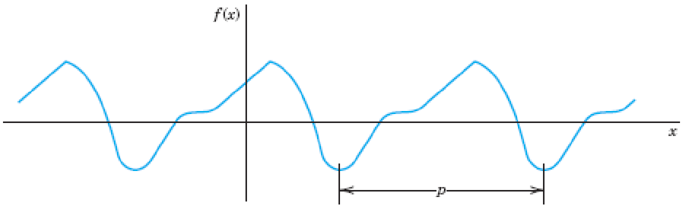


Fig. 258. Periodic function of period p

Fourier Analysis

All these functions have the period 2*pi. They form the so-called **trigonometric system.**

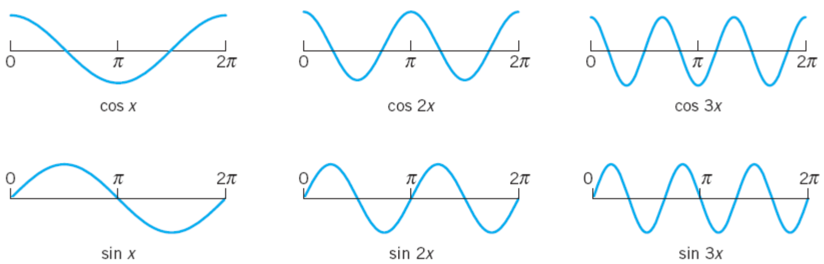


Fig. 259. Cosine and sine functions having the period 2π (the first few members of the trigonometric system (3), except for the constant 1)

Fourier Analysis

Now suppose that **f(x)** is a given function of period **2pi** and is such that it can be **represented by a series (3), that is, (3) converges.** Then, using the equality sign, we write

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx) \quad (3)$$

The question is:

What are the coefficients a_0, a_n and b_n??

Fourier Analysis

Now we are going to obtain the coefficients:

If we integrate both sides of Equation 3 and assume that it's permissible to integrate the series term-by-term, we get:

$$\int_{-\pi}^{\pi} f(x)\,dx = \int_{-\pi}^{\pi} a_0\,dx + \int_{-\pi}^{\pi} \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)\,dx$$

$$= 2\pi a_0 + \sum_{n=1}^{\infty} a_n \int_{-\pi}^{\pi} \cos nx\,dx + \sum_{n=1}^{\infty} b_n \int_{-\pi}^{\pi} \sin nx\,dx$$

Note that:

$$\int_{-\pi}^{\pi} \cos nx\,dx = \frac{1}{n}\sin nx\Big]_{-\pi}^{\pi} = \frac{1}{n}[\sin n\pi - \sin(-n\pi)] = 0$$

Similarly, $\int_{-\pi}^{\pi} \sin nx\,dx = 0$.

Fourier Analysis

So:

$$\int_{-\pi}^{\pi} f(x)\,dx = 2\pi a_0$$

Therefore:

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx \qquad (4)$$

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

To determine a_n for $n \geqslant 1$ we multiply both sides of Eq (3) by $\cos mx$

$$\int_{-\pi}^{\pi} f(x)\cos mx\,dx = \int_{-\pi}^{\pi}\left[a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)\right]\cos mx\,dx$$

$$= a_0\int_{-\pi}^{\pi}\cos mx\,dx + \sum_{n=1}^{\infty} a_n \int_{-\pi}^{\pi}\cos nx\cos mx\,dx + \sum_{n=1}^{\infty} b_n \int_{-\pi}^{\pi}\sin nx\cos mx\,dx$$

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Fourier Analysis

it's not hard to show that

(a) $\int_{-\pi}^{\pi}\cos nx\cos mx\,dx = 0 \qquad (n \neq m)$

(b) $\int_{-\pi}^{\pi}\sin nx\sin mx\,dx = 0 \qquad (n \neq m)$

(c) $\int_{-\pi}^{\pi}\sin nx\cos mx\,dx = 0 \qquad (n \neq m \text{ or } n = m).$

Hint:

$$\int_{-\pi}^{\pi}\cos nx\cos mx\,dx = \frac{1}{2}\int_{-\pi}^{\pi}\cos(n+m)x\,dx + \frac{1}{2}\int_{-\pi}^{\pi}\cos(n-m)x\,dx$$

$$\int_{-\pi}^{\pi}\sin nx\sin mx\,dx = \frac{1}{2}\int_{-\pi}^{\pi}\cos(n-m)x\,dx - \frac{1}{2}\int_{-\pi}^{\pi}\cos(n+m)x\,dx.$$

$$\int_{-\pi}^{\pi}\sin nx\cos mx\,dx = \frac{1}{2}\int_{-\pi}^{\pi}\sin(n+m)x\,dx + \frac{1}{2}\int_{-\pi}^{\pi}\sin(n-m)x\,dx = 0 + 0.$$

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Fourier Analysis

Then, solving for a_n, we get:

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx \qquad n = 1, 2, 3, \ldots \tag{5}$$

Similarly, if we multiply both sides of Equation (3) by $\sin mx$ and integrate from $-\pi$ to π,

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx \qquad n = 1, 2, 3, \ldots \tag{6}$$

Fourier Analysis

THEOREM 1 Let f be a piecewise continuous function on $[-\pi, \pi]$. Then the **Fourier series** of f is the series

$$a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \tag{7}$$

where the coefficients a_n and b_n in this series are defined by

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx \tag{8}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

and are called the **Fourier coefficients** of f.

Questions? Discussion? Suggestions ?

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